Clustering in Data Streams: Improving BIRCH

Project: Practical Theory for Clustering Algorithms

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The k-means Problem

- Given a point set $P \subseteq \mathbb{R}^d$,
- compute a set $C \subseteq \mathbb{R}^d$ with $|C| = k$ centers
- which minimizes $\text{cost}(P, C)$

$$= \sum_{p \in P} \min_{c \in C} ||c - p||^2,$$

the sum of the squared distances.

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- arbitrary order
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- In Practice: BIRCH as a well-known heuristic
- In Theory: Coreset Theory

Clustering Feature

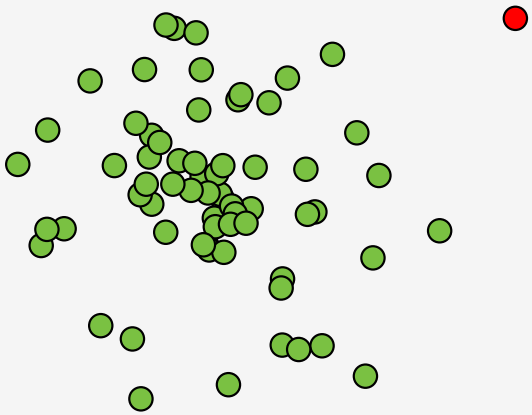
Fact

The sum of the squared distances satisfies the equation

$$\sum_{p \in P} ||p - z||^2 = \sum_{p \in P} ||p - \mu||^2 + |P| ||\mu - z||^2$$

where μ is the centroid of P.

Clustering Feature



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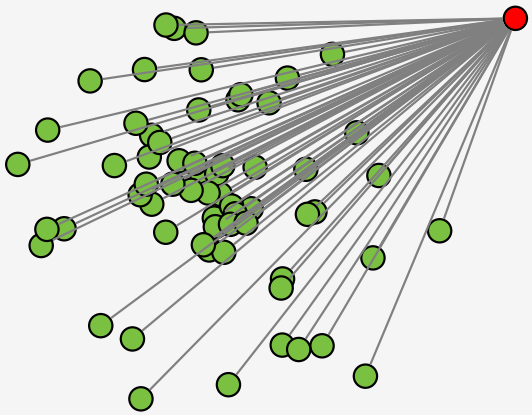

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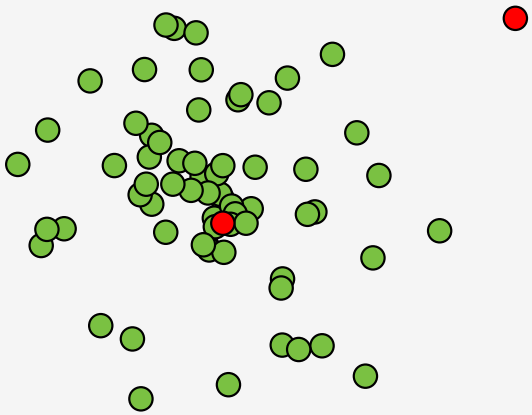

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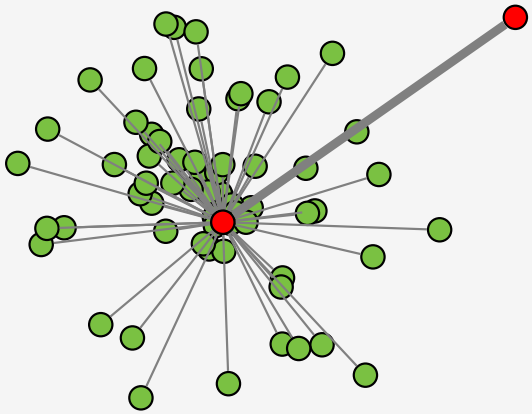

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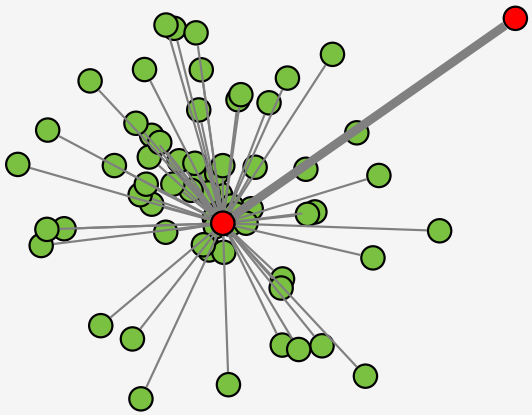

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- Simply store $|P|, \sum_{p \in P} p$ and $\sum_{p \in P} ||p||^2$
- $\sum_{p \in P} ||p||^2 = \sum_{p \in P} ||p - \mu||^2 + |P|||\mu||^2$
- $\sum_{p \in P} ||p - z||^2 = \sum_{p \in P} ||p||^2 - |P|||\mu||^2 + |P|||\mu - z||^2$

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- BIRCH searches for the 'closest' CF according to

$$\sum_{q\in(S\cup\{p\})}\left(q-\frac{\sum_{q\in(S\cup\{p\})}q}{|S|+1}\right)^2-\sum_{q\in S}\left(q-\frac{\sum_{q\in S}q}{|S|}\right)^2$$

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- p is added to CF representing subset S^* if $\sqrt{\frac{\sum_{p\in(S^*\cup\{p\})}(q-\mu_S)^2}{|S|+1}}\leq T$ for a given threshold

Coreset Theory

Coresets

Given a set of points P, a weighted subset $S \subset P$ is a (k, ϵ)-coreset if for all sets C of k centers it holds

$$|\text{cost}_w(S, C) - \text{cost}(P, C)| \leq \epsilon \, \text{cost}(P, C)$$

where $\text{cost}_w(S, C) = \sum_{p \in S} \min_{c \in C} w(p)||p - c||^2$.

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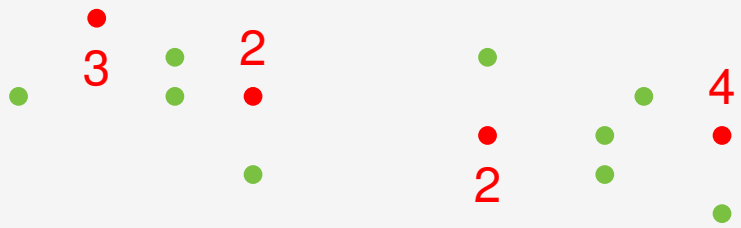

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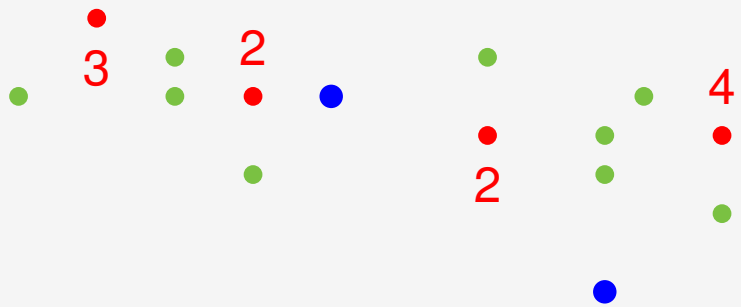

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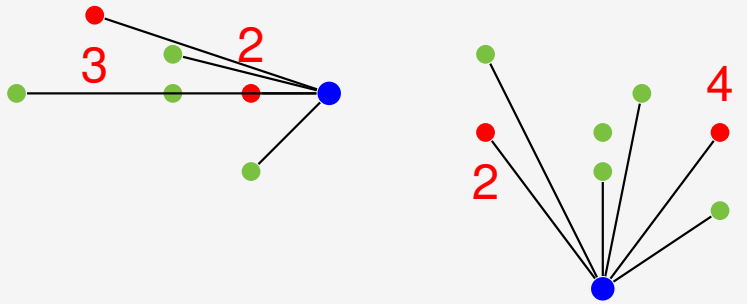

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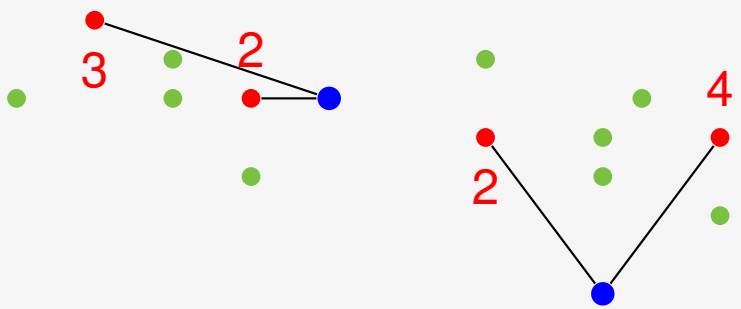

Coreset constructions

- '01: Agarwal, Har-Peled and Varadarajan: Coreset concept
- '02: Bădoiu, Har-Peled and Indyk: First coreset construction for clustering problems
- '04: Har-Peled and Mazumdar, Coreset of size $\mathcal{O}(k\epsilon^{-d}\log n)$, maintainable in data streams
- '05: Frahling and Sohler: Coreset of size $\mathcal{O}(k\epsilon^{-d}\log n)$, insertion-deletion data streams
- '06: Chen: Coresets for metric and Euclidean k-median and k-means, polynomial in d,n and ϵ^{-1}
- '07: Feldman, Monemizadeh, Sohler: weak coresets, poly(k, ϵ^{-1})
- '10: Langberg, Schulman: $\tilde{\mathcal{O}}(d^2k^3/\varepsilon^2)$
- '11: Feldman, Langberg: $\mathcal{O}(dk/\varepsilon^2)$

Merge & Reduce: Coreset Construction $\rightsquigarrow$ Streaming Algorithms.

StreamKM++

- also an outcome of this SPP
- practical k-means streaming algorithm
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- better solutions than BIRCH, but slower

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Motivations to Improve BIRCH

- Analyzable BIRCH is valuable
- Might outperform both StreamKM++ and BIRCH
- Hope of keeping good practical properties

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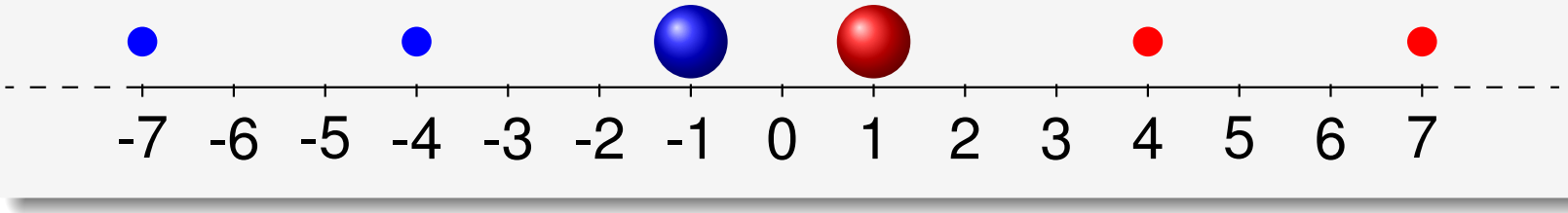

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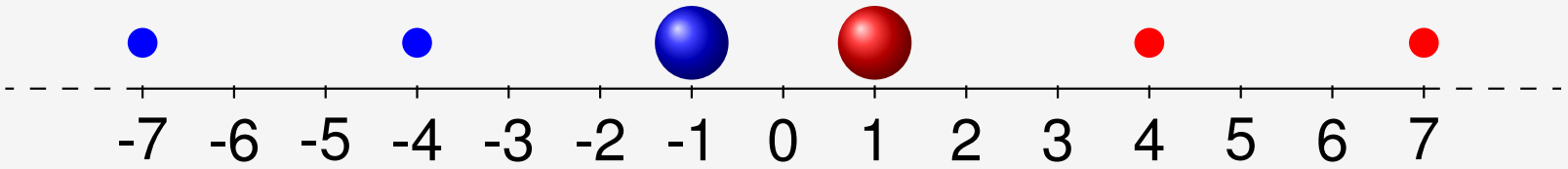

Lemma

Depending on the threshold T, BIRCH either needs $\Omega\left(\frac{|P|}{d}\right)$ CFs or it computes no constant factor approximation.

(For a generalized example)

Lessons from Coreset Theory

- Base insertion decision on induced error
- Error can be bound if the clustering cost of a CF is small
- Use packing arguments to bound number of CFs

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Insertion of a point

1. Levels of clustering features, Start at top level
2. Search for closest CF
3. Point has to lie within the radius of the CF
   (Radius decreases by constant factor per level)
4. Add point if clustering cost of CF stays below $\frac{f(\varepsilon)}{k \cdot g_i} \cdot OPT$
5. If insertion fails, open a new CF or go one level down

Small Change, Huge Effect

Analysis: Quality

- Inspired by known coreset constructions
- Distinguish between points close to optimal centers ($\rightarrow$ packing argument)
- and far away centers (error neglectable to clustering cost)

Analysis: Number of CFs

1. Bound number of levels:
   Constant Factor between Radii $\rightarrow$ number of points until full doubles $\rightarrow$ logarithmic in the number of points
2. Number of full CFs:
   can be bound by lower bound on their clustering cost
3. Two types of non-full CFs:
   Children of full CFs ($\rightarrow$ bound carries over)
4. and non-full CFs on the first level ($\rightarrow$ packing argument)

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And if *OPT* is not known?

- dynamically increase threshold
- Analysis still works, but gets more involved

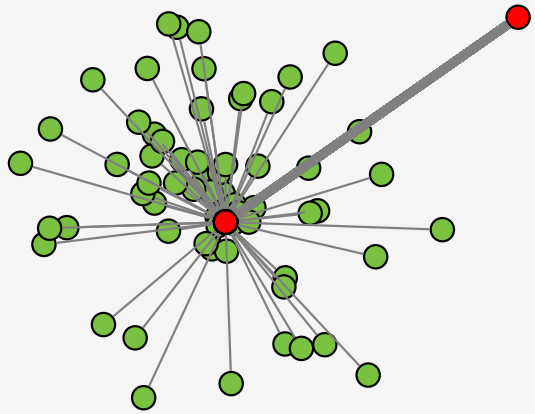
Theorem

The modified BIRCH algorithm computes a (k, ε)-coreset if *OPT* is known and can be modified for the case that *OPT* is not known. The size of the coreset is

$$\mathcal{O}\left(\left(\frac{k}{f(\varepsilon)}\right)^{d} + 2^{c \cdot d} \cdot \frac{k}{f(\varepsilon)} \cdot \log n \log^2 \log n\right).$$

And what is still missing. . .

. . . is the experimental analyses. This is the next step :-)



Thank you for your attention!